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Carderock Division, Naval Surface Warfare Center

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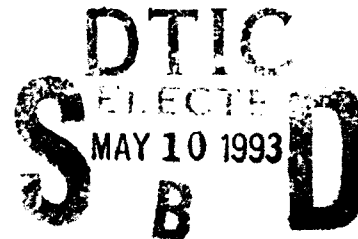# Unsteady Vortex Loop/Dipole Theory Applied to the Work and Acoustics of an Ideal Low Speed Propeller

by:
Earl Quandt

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Unsteady Vortex Loop/Dipole Theory Applied to the Work and Acoustics of an Ideal Low Speed Propeller

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CONTENTS

FIGURES

DTIC QUALITY INSPECTED 5

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NOMENCLATURE

Symbol	Units	Definition
A	L^2	Surface Area
A_n	F/L^2	Fourier Coefficient
a_i	L	Radius of sphere
B	——	Number of blades
C_L	——	Lift Coefficient
c	L/T	Speed of Sound
D	L^4/T	Dipole strength
d	L^3/T	Dipole strength per length
F	F	Force
G	L^4/T	Dipole strength
h	L	Length of bound vortex
h_T	L^2/T^2	Total enthalpy
I	FT	Impulse
i, j	——	Unit vector
J	——	Bessel function
k	1/L	Acoustic wave number
L	L	Chord length
ℓ	L	Length or integer index
M	——	Mach number
$\dot{m}$	M/T	Mass flow
m, n	——	Integer index
n_s	——	Surface normal vector
P	FL/T	Power
p	F/L^2	Pressure
R	L	Radius on rotating blade
r	L	Distance between points
s	L	Distance
$\mathbf{T}$	F	Thrust
$\mathbf{T}$	FL	Torque
t	T	Time
$\bar{t}$	L	Mean blade thickness
u_r	L/T	Fluid velocity in radial direct on
V	L/T	Velocity in absolute frame
v_β	L/T	Velocity in circumferential direction
$\mathbf{V}$	L^3	Volume
W	L/T	Velocity Relative to blade
x, y, z	——	Coordinate axes

NOMENCLATURE

Greek	Units	Definition
β	——	Angle with respect to a coordinate
Γ	L^2/T	Vortex strength
γ	——	Angle of lift force to rotor plane
Δ	L^3	Displacement
δ	——	Small difference
η	——	Fraction of power radiated acoustically
ξ	——	Mach number — angle parameter
φ	——	Spherical surface angle
ϕ	L^2/T	Velocity potential
ρ	M/L^3	Fluid density
ζ	$1/T$	Fluid vorticity
θ	——	Angle in circumferential direction
Ω	$1/T$	Shaft rotation rate

Subscripts	Definition
a	Acoustic
B	Blade
D	At the propeller disc
i	Isolated element
I	Inner radius of element
L	Lift
n	Harmonic integer
o	From origin, to the observer
s	Source or surface
T	Thickness
x, z	Referred to the x or z direction
θ	Referred to circmferential direction

ABSTRACT

The generation of work and acoustic effects in a fluid by the action of a turbomachine is examined using an unsteady potential flow model of a low-speed open propeller. Individual blade thickness and lift characteristics are represented as radially distributed dipole and vortex loop singularities that rotate with the shaft. For multiple blades, the summed potentials of the vortex loops alone account for the work input and the associated three-dimensional energy distribution in the wake. With slight fluid compressibility, tonal acoustic signals are produced by the motion of both singularities, with the lift term typically dominating. Although the acoustic power input also increases with additional blades, the dipole nature produces in the far field a large apparent reduction due to cancellation. By relating both singularities to equivalent orginating impulses, it can be seen how these moving geometry-based functions also result in an unsteady potential field that satisfies automatically the momentum and energy conservation laws.

ADMINISTRATIVE INFORMATION

INTRODUCTION

The conditions for potential flow are used frequently for the preliminary analysis of the velocity fields created by the motion of solid surfaces through a fluid, Hess,[1] and almost always for examining many of the accompanying acoustics, Howe.[2] While such analyses are often made separately, and in a frame of reference attached to the body, it is sometimes useful to consider an unsteady formulation where the body moves. This approach provides both the work and acoustics in the same solution. Although it is more difficult to formulate functions that vary with space and time, this approach is useful for turbomachines since it emphasizes the fundamental need for unsteady fluid motion in the energy transfer process, Dean,[3] and leads smoothly to the origin of the tonal sound field. Fortunately, for multiple surfaces moving uniformly through the absolute frame, the techniques of periodic functions are available, and the additive property of elemental potential flow solutions can be used to isolate some of the general features of these complex repeating flow fields.

For flows that are inviscid, irrotational, and incompressible, Lamb[4] shows that a scalar velocity potential, ϕ, will exist as a solution to the Laplace representation of the continuity equation. For solid surfaces moving through a fluid, such as a blade, the solutions to this equation are a function only of the geometric boundaries at any instant. The condition used for the solution is that the relative velocity normal to any solid surface is zero.

$$W_n = \vec{n}_s \cdot \vec{W} = \vec{n}_s \cdot (\vec{V} - \vec{V}_B) = 0 \qquad \vec{V} = \vec{\nabla}\phi \qquad (1)$$

In addition, when there is a sharp trailing edge, as for a lifting blade, it is necessary to use a Kutta condition at that point to require the leaving velocities to merge smoothly. In principle all these conditions can be met by a Green's integral technique that sums a distribution of elemental solutions over all blade surfaces to produce a total velocity potential at any point in the fluid. Herein it will be preferred to use dipole element solutions to represent blade thickness and a bound and shed vortex loop system to account for blade lift and wake effects.

In an early study of how the unsteady velocity potential applied to moving blades, Preston[5] showed that a repeating system of two dimensional vortex singularities provided a time-varying velocity potential that gave the correct blade force and velocity change for a fluid convecting across a simulated blade row. When used in the unsteady Bernoulli equation;

$$\vec{\nabla} \left(\frac{\partial \phi}{\partial t} \right) = - \vec{\nabla} \left(\frac{p}{\rho} + \frac{V^2}{2} \right) = - \vec{\nabla}(h_T) \tag{2}$$

the time and space variation of the potential for the moving vortex row produced the appropriate fluid enthalpy rise across the row, and therefore the work, for both axial and centrifugal geometries. About the same time Van de Vooren and Zandbergen[6] simulated the acoustics of an open propeller by using vortex loop and dipole singularities moving in a helical path. More recently Hanson[7] has expanded on the aeroacoustics of Goldstein[8] to present a unified potential theory based on delta functions for the work and acoustics of aircraft propellers. In the marine field Kerwin[9] has developed several Green's integral techniques for analyzing the steady and unsteady (Kinnas[10]) flow fields of water propulsors, while Blake[11] has summarized many applicable acoustic principles.

From a mathematical standpoint Lighthill[12] has shown that it is the nature of distributed elemental potential solutions to produce, at a distance, a flow field that is characteristic of one larger equivalent singularity. In this way Lighthill developed the following useful formula for the external force vector with which the surroundings act on the fluid in order to represent the motion of a solid body.

$$\vec{F}(t) = \frac{\rho}{2} \frac{\partial}{\partial t} \iiint (\vec{x} \times \vec{\zeta}) \, d\forall + \rho \Delta \frac{d\vec{V}_B}{dt} \tag{3}$$

For a rotating blade, this equation allows definition of the torque arising from shaft rotation, and thus the work, as a function of the time rate of change of two momentum-like terms. Clearly this work input must also appear as a change in the total enthalpy of the through-flow in order to satisfy the energy conservation equation. Lighthill's formula is compatible with this requirement since it considers the total vortex loop system set up in the wake of a lifting surface and may be thought of as the three-dimensional extension of Preston's analysis.

Turning to the acoustic effect from a rotating blade, Lamb shows that for a slightly compressible fluid the velocity potentials due to the surface motion are augmented by a term involving their rate of change, and that this total potential function propagates away from the source at the speed of sound. Again, in the far-field the details of the blade surface are smoothed when the Mach number of the motion is small, such that the source appears as one characteristic singularity. The remote acoustic pressure caused by a force applied near the origin is commonly represented by the Curle[13] equation for propagation in a free field.

$$p_a(\vec{r}, t) = \frac{\vec{n}_o}{4\pi r_o c} \cdot \left[\frac{d\vec{F}(t)}{dt} \right]_{t-r_o/c} \tag{4}$$

Here the force is exactly that from Lighthill's formula, and the square brackets are used to represent the acoustic convention that the function must be evaluated at a retarded time based on the distance from the source to the observer. For the blade of an ideal turbomachine operating in a uniform flow, this force will be constant in magnitude and variable in direction.

For both the work and acoustic behavior it thus appears that the elementary singularity functions that are summed in potential flow to meet the surface boundary conditions of a rotating blade must also be relatable to forces applied from the surroundings and work appearing in the fluid. In fact this is fundamental to potential flow theory as developed by Lamb, and emphasized by Lighthill who shows that at any instant both the vortex loop and dipole velocity potentials are each the result of vector impulses applied to the fluid. As a further convenient equivalence, Wu[14] has shown how the velocity potentials of a small vortex loop at the origin, and a dipole at the origin are related.

$$\phi(\vec{r}) = \vec{n}_0 \cdot \frac{\vec{D}}{4\pi r_0^2} = \vec{n}_0 \cdot \frac{(\Gamma A)}{4\pi r_0^2} \vec{n}_s = \vec{n}_0 \cdot \frac{\vec{I}/\rho}{4\pi r_0^2} \tag{5}$$

Here, I is the common impulse needed to create from rest either singularity. Therefore the external force needed to move a blade from one point to the next can be determined from the rate of change of the impulses that simulated the flow fields at the two points, i.e.,

$$\vec{F}(t) = \sum \frac{d\vec{I}}{dt} \tag{6}$$

Recalling the Lighthill equation, its separate terms are due to such boundary impulses, and are thus related implicitly to momentum and work effects in the fluid.

It is the purpose of this paper to explore how the unsteady forms of the vortex loop and dipole singularity functions act to create the dynamic potential field of a turbomachine. In doing this, local details will be neglected in favor of emphasizing the basic nature of these functions and how their sum reflects the overall influence of a rotating blade. The geometry of a stationary open axial flow propeller, with no adjacent surface, will be used as the simplest arrangement in which to illustrate these unsteady effects. The complete distribution of singularity strengths and directions will be assumed known from a Green's Integral solution, or estimatable, and these will be considered to be grouped in radial blade slices. It will be assumed that the blades are equal and equally spaced, sufficiently small that they act in isolation, having steady rotation, uniform inflow and low Mach number. The development of the steady thrust and torque as well as the form of the fluid total enthalpy change will be expressed in terms of the unsteady singularities. Secondly, the response of a slightly compressible fluid to the same singularities will be examined to show how the acoustic solution is formed and why it differs from the incompressible solution. Some conclusions will be made concerning the additional insight provided by an unsteady three-dimensional potential analysis.

TURBOMACHINE WORK

The basic purpose of a turbomachine is to convert shaft work into a change of the energy of the fluid moving through the machine. As noted in the Introduction, for incompressible potential flow, the blade surfaces of the turbomachine perform this work. Their effect on the flow field is characterized, through the Green's Integral solution, by a surface distribution of dipoles and vortex loops that represent the thickness and lift properties of each blade. The additive property of potential flow solutions makes it possible to group these singularities in radial segments and to study, at some distance, the field contribution of each, as long as the components still satisfy the geometric boundary conditions for the multiple rotating blades. Thus, as a blade rotates in a uniform in-flow, some external force must be applied to each

radially positioned singularity that is equal to the rate of change of the surface impulse associated with that singularity. The work input to the fluid is the integral of the scalar product of these forces with the local blade velocity. From an overall energy balance, with steady conditions at the boundaries, the power input from B equal blades must equal the gain in total enthalpy of the mass flow through the machine, i.e.,

$$\text{Shaft Work} = B \sum_{i=1}^{N} \vec{F_i} \cdot \vec{V}_{B_i} = B\Omega \sum_{i=1}^{N} \mathcal{T}_i = \iint h_T \, \rho \vec{V} \cdot \vec{ds} = \dot{m}_D \Delta h_T \tag{7}$$

Here, the total enthalpy change can be determined by applying the unsteady Bernoulli equation between any two positions in the flow.

Figure 1 illustrates the geometry to be examined, highlighting the fact that each blade will be treated individually and the effects summed. In the following sections the unsteady forms of the small dipole and vortex loop singularities will be examined separately to evaluate their effect on the velocity potential and on the result of the uniform rotation of these singularities, as viewed from a fixed frame of reference.
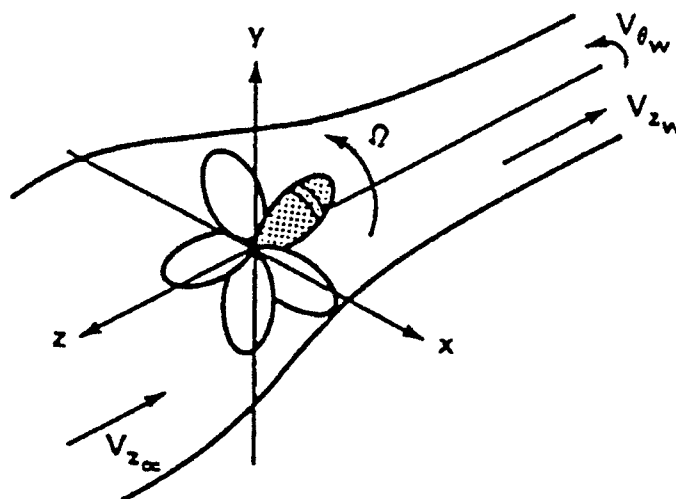
Fig. 1. Flow field geometry for open propeller.

Blade Thickness Work

Considering a symmetric blade at zero lift, the Green's integral solution simulates the blade volume by a set of dipoles distributed along the chord and the radius, and oriented to oppose the relative flow. Taking this effect at a distance, these dipoles may be lumped into smaller groups distributed radially at the mass center of momentum for each radial segment. It is known that steady flow over a fixed dipole does not require a force, but in this case the dipole singularity is rotating so that it has varying direction and constant strength. It will therefore be useful to examine this unsteady behavior of the thickness dipoles, so as to illustrate certain aspects of the solution.

Looking at an isolated dipole singularity, the instantaneous velocity potential around any arbitrary origin is:

$$\phi_{T_i}(\vec{r}) = -\frac{\vec{n}_0}{4\pi r^2} \cdot \vec{G}_{T_i} \tag{8}$$

Using the Green's surface integral theory, Lighthill shows that this total dipole strength may be thought of as composed of two parts;

$$\vec{G}_{T_{i_1}} = \vec{I}_{T_i}/\rho = -\iint \phi_i \, \vec{n}_s \, ds$$

$$\vec{G}_{T_{i_2}} = \Delta_i \vec{V}_i = \iint \left(\vec{s} \cdot \frac{\partial \vec{\phi}_i}{\partial n}\right) \vec{n}_s \, ds \tag{9}$$

Here the vector velocity represents the motion of the fluid displaced by the blade volume. Thus the force needed to cause the motion of an "effective" isolated volume is:

$$\vec{F}_{T_i}(t) = \frac{d\vec{I}_{T_i}}{dt} + \rho\Delta_i \frac{d\vec{V}_i}{dt} = \rho \frac{d\vec{G}_{T_i}}{dt} \tag{10}$$

Using the dipole velocity potential in the surface integral for the first term gives;

$$\vec{G}_{T_{i_1}} = \frac{1}{\rho} \int_0^t \left(\iint p\,\vec{n}_s \, ds\right) dt = \frac{\Delta_i}{2}\,\vec{V}_i(t) \tag{11}$$

which is recognized as both an impulse and the momentum of the "added mass".

Interestingly, this impulsive part of the dipole strength can also be interpreted in terms of a volume integral of the moment of vorticity associated with the motion. To appreciate this it is necessary to write the circumferential velocity on the surface of the effective sphere as

$$v_{\beta_i} = \frac{1}{r} \frac{\partial \phi_i}{\partial \beta} = \frac{\vec{V}_i(t)}{2} \sin\beta \tag{12}$$

This circumferential velocity can be simulated with a shear layer distribution of vortex rings in the surface of the apparent sphere with surface normal in direction V_i. Taking the following volume integral over all the region yields values only for the radius a.

$$\frac{1}{2} \iiint (\vec{x} \times \vec{\jmath}) d\Psi = \frac{\vec{n}_i}{2} \int_0^\pi a \sin\beta \left(\int_0^{2\pi}\int_0^a |\zeta|a \sin\beta \, d\varphi dr\right) d\beta = \frac{\Delta_i}{2}\,\vec{V}_i(t) \tag{13}$$

Therefore the Lighthill formula for the force to move a solid body is directly related to the force needed to change the spatial location of the velocity potential of an isolated dipole. The total dipole strength and direction at any radius is equivalent to;

$$\vec{G}_{T_i}(t) = \frac{3\Delta_i}{2}\,\vec{V}_i(t) = \frac{1}{2}\iiint(\vec{x}\times\vec{\varsigma}_i)\,d\vee + \Delta_i\vec{V}_i(t) \tag{14}$$

When the dipole is of constant strength, and rotating uniformly, the vector velocity is;

$$\vec{V}_i(t) = \vec{\Omega}\times\vec{r}_i = \Omega R_i\,(-\vec{i}\,\sin\Omega t + \vec{j}\,\cos\Omega t) \tag{15}$$

the vector acceleration is;

$$\frac{d\vec{V}_i(t)}{dt} = -\vec{\Omega}\times(\vec{\Omega}\times\vec{r}_i) = -\Omega^2 R_i\,(\vec{i}\,\cos\Omega t + \vec{j}\,\sin\Omega t) \tag{16}$$

and, as for any rotating body, the force due to acceleration is always radially inward and normal to the velocity. Therefore, as expected, the incompressible work associated with the rotation of the dipole that simulates the blade volume is zero since the scalar product of force and velocity is zero.

Blade Lift Work

The velocity potential created in the fluid by a lifting surface is more complex than that of the volume, but also with certain important equivalencies. This case will be examined by considering radial segments of a blade of zero thickness rotating steadily at some constant angle of attack to the relative flow. The basic singularity that allows the boundary conditions of zero normal relative velocity on the surface and equal parallel velocities at the trailing edge is the vortex loop. This three-dimensional potential flow function is initiated from a vortex line in the surface of the blade with length equal to the radial increment and direction normal to the relative velocity. Since vortex lines cannot end in the fluid, each end of the bound vortex is connected to shed vortex lines that convect with the flow, and are of direction consistent with joining to the bound vortex. As shown in Fig. 2, the shed vortex lines wrap helically around the axis of rotation, and at some far downstream point connect to a starting vortex such that the whole system makes a closed loop.
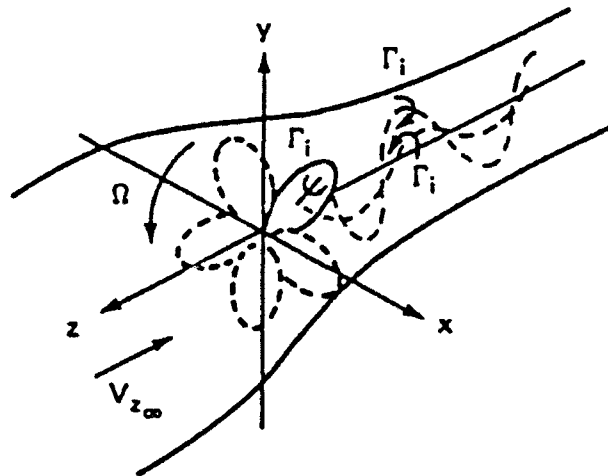

Fig. 2. Pattern of double helix vortex loop in the wake of one blade.

As a result of this vortex loop, two unique flow features appear. First, there is a net value for the line integral of velocity around any closed path that includes a vortex line.

$$\oint \vec{V} \cdot d\vec{\ell} = \Gamma \tag{17}$$

Close to a vortex line, the flow may be considered two dimensional such that a vortex velocity potential referred to the axis of the line and compatible with the above is:

$$\phi_L (r,\theta) = \frac{\Gamma}{2\pi} \theta \tag{18}$$

In the wake, the pair of shed vortex lines are of opposite rotation and any large closed path integral that includes both will have a value of zero. Nevertheless this double helix vortex system in the wake is of major importance since it can also be considered as a long chain of small vortex loops. As noted in the Introduction, the velocity potential of a small vortex loop is, at a distance, the same as that of a dipole centered in the loop with direction equal to the loop surface normal. Thus the vortex system shed from the rotating lifting blade represents also a helical distribution of dipoles convecting along the axis of rotation. The strength of the dipoles per unit length of the wake is given by

$$\vec{d} = \frac{\delta \vec{D}}{\delta \ell} = \frac{\Gamma \delta A \vec{n}_s}{\delta \ell} = \frac{d\vec{G}_L}{d\ell} \tag{19}$$

A second feature of the potential flow field produced by the vortex singularity is that the local velocity potential can be ambiguous if a path is followed around one vortex line. Although the total strength of the vortex is defined by one complete circuit, the velocity potential function for the vortex line appears to increase with every sequential 2π circuit. In order to prevent this uncertainty, permissible regions having continuity of the velocity potential are restricted therefore to paths outside of the vortex loop. To accomplish this mathematically, the vortex loop area is taken as a boundary surface of the flow field over which the velocity is continuous, but across which the velocity potential experiences a step jump. Thus to return to the proper starting value the discontinuity must be:

$$\Delta \phi_L = -\Gamma \tag{20}$$

From a distance, the effect of a lifting surface on the fluid can then be viewed either as a continuous injection of dipoles or as an expanding vortex loop. Choosing the latter, it is helpful to note that a vorticity distribution can be simulated by a singular vortex circulation whose strength is determined by integration over a surface normal to the vorticity. Therefore it is also possible to characterize this fluid motion using the Lighthill integral of the moment of vorticity.

$$\vec{F}_L(t) = \rho \frac{d\vec{G}_L}{dt} = \frac{\rho}{2} \frac{\partial}{\partial t} \iiint (\vec{x} \times \vec{\varsigma}_L) \, d\Psi \approx \rho \Gamma Wh\vec{n}_L \tag{21}$$

Thus the vorticity moment integral is a three-dimensional expression of the ubiquitous two-dimensional equation for the force due to flow over a line vortex.

Behavior at Rotor Disc

Although the integral of Equation (21) includes the entire volume, the point of introduction of the impulses, and thus the force, is on the blade surface. Integration of the detailed pressure over a blade surface must also produce a total force consistent with the above volume integral. In any event, this total force will be steady and, except for induced drag, in a direction normal to the local relative velocity. Therefore, looking at the work and power transfer for any radial element

$$dw_{L_i} (r) = F_{L_i} (r) \sin \gamma_i (r) \, \overline{R}_i \, d\theta$$

$$P_{L_i} (r) = \frac{dw_{L_i}}{dt} = F_{L_i} \sin \gamma_i \, \overline{R}_i \Omega \tag{22}$$

Considering a short radial blade span over which the local vortex singularity strengths have been summed to one equivalent radial vortex line, the resulting vortex distributions from several blades are shown in Fig. 3. Applying the unsteady Bernoulli equation in the flow requires:

$$\overline{\nabla} \left(\frac{\partial \phi_i}{\partial t} \right) = - \overline{\nabla} \left(\frac{p}{\rho} + \frac{V^2}{2} \right)_i = - \overline{\nabla} (h_{T_i}) \tag{23}$$
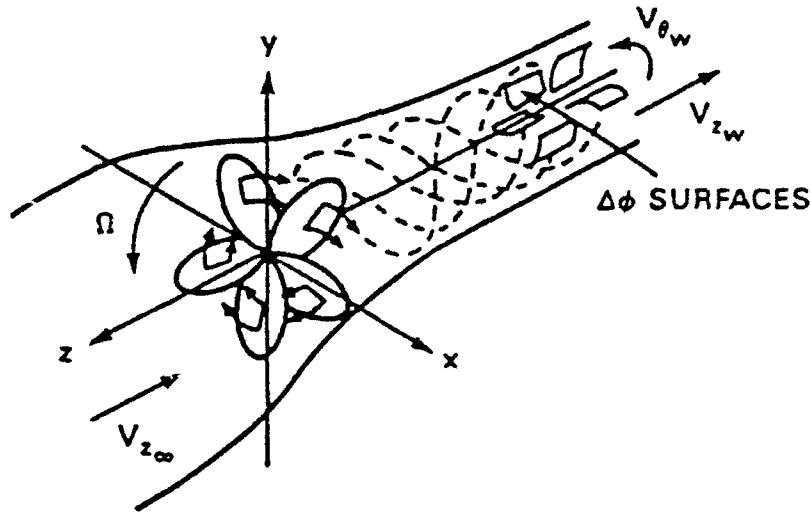
Fig. 3. Structure of vortex loops downstream of multiple blades.

At some fixed point in the wake of the ith radial section, the velocity potential will jump by $\Delta \phi_i$ for each blade passage. Thus if a mean total enthalpy change at any point in the wake is considered, it should be given by the "average" rate at which vortex loops pass through that point referred to fluid not effected by the propeller disc. For B equal blades:

$$\overline{\Delta h}_{T_i} = - \frac{\overline{\Delta \phi}_i}{\Delta t} = \frac{B \Omega \Gamma_i (r)}{2\pi} = (\Delta V_\theta (r) \, V_B (r))_D \tag{24}$$

Here the local change in angular velocity at the propeller disc has been taken as the two-dimensional solution for a circumferentially uniform distribution of vortices, i.e.,

$$\Delta V_\theta(r) = \frac{B\Gamma_i(r)}{2\pi r_i} \qquad (25)$$

Equation (24) is the classic Euler turbomachine average head rise equation, and is the same result as for the infinite blade number approximation, where the local details are not as important as the overall behavior. Using this in an actuator disc model, the total enthalpy rise is experienced at the disc as a jump in static pressure so that the force component in the axial direction, or the thrust on the shaft, is obtained from a summation of axial forces over the disc.

$$\mathcal{F} = \sum_{i=1}^{N} A_{D_i} \Delta p_{D_i} = \sum_{i=1}^{N} \frac{\rho\Omega}{2} B\Gamma_i (R_o^2 - R_I^2) = \sum_{i=1}^{N} \rho B\Gamma_i \vec{V}_{B_i} (R_o - R_I)_i \qquad (26)$$

In a similar manner, the torque is the change in the mass flux of angular momentum across the disc for all the radial segments.

$$\mathcal{T} = \sum_{i=1}^{N} \dot{m}_i \Delta (R_i V_{\theta_i}) = \sum_{i=1}^{N} \rho V_{zD_i} (R_o - R_I)_i \, \overline{R}_i B\Gamma_i \qquad (27)$$

With B blades, Equation (27) can be used to give the overall energy balance of Equation (7) for any stream tube passing through the rotor disc.

In engineering terms it is helpful to characterize the circulation for a high aspect ratio blade by the equivalent lift coefficient:

$$F_{L_i} = \rho\Gamma_i W_i h_i = C_{L_i} \rho \frac{W_i^2}{2} h_i L_i \qquad (28)$$

Thus the local average blade circulation, i.e., the strength of the vortex singularity at any radius, can be approximated by

$$\Gamma_i = \frac{C_{L_i} W_i L_i}{2} \qquad (29)$$

where the lift coefficient usually varies from 0 to 1. In practice it is not possible to maintain a high circulation near the axis of rotation, so in general there will be a continuous variation from zero at the hub to zero at the tip. Through Equation (24) this produces radial variations in swirl angle and total enthalpy downstream of the disc.

Behavior in the Wake

For the far wake, the shed vortex system will have spiralled around the axis of rotation many times, and isolated blade effects will be smoothed in the circumferential direction. In this case the velocity will be steady and there will be an "apparent vorticity" in the fluid having a direction following the particle pathlines where:

$$\vec{\zeta}_{L_i} = \frac{B\Delta\Gamma_i}{2\pi r_i \, \Delta r_i} \, \vec{n}_{L_i}(z) \qquad (30)$$

For steady inviscid rotational flow the momentum equation requires

$$\vec{\nabla} (h_T) = - \vec{\zeta} \times \vec{V} \tag{31}$$

where for this case both the vorticity and the absolute velocity vectors are in cylindrical surfaces. Since the local "apparent vorticity" changes sign in moving from an inner to an outer radius as the circulation distribution peaks, the total enthalpy computed from the vortex loop distribution is also seen to peak at some mid-radius. Since the static pressure at the wake boundary is always ambient, the circumferential velocity components produce lower static pressures toward the centerline.

To summarize, the three dimensional unsteady effect of a rotating vortex loop singularity is to modify the flow field in the wake of one or more blades in such a way as to account for the torque, thrust and work of the turbomachine. It should be pointed out that if the blades were stationary, a vortex system would still develop, but there would be no work input and no change in the fluid total enthalpy, such that kinetic energy changes would be balanced by static pressure changes.

TURBOMACHINE ACOUSTICS

As a moving blade acts on an unbounded fluid that is slightly compressible, vortex loop/dipole singularity functions are still used to satisfy the surface boundary conditions of the Green's integral solution. However because of the small compressibility, additional terms arise in the velocity potential to account for the time rate of change of the strength, and for the acoustic propagation to the far field. To examine the acoustic effect it will be most convenient to express the behavior of all singularities in terms of the dipole representation of blade motion. Thus for a general dipole at the origin for which both strength and direction are changing, the resulting velocity potential at any remote point in a slightly compressible fluid is given by.

$$\phi_a(\vec{r},t) = - \vec{n}_o \cdot \left[\left(\frac{G(t)}{4\pi r^2} + \frac{\mathcal{J}'(t)}{4\pi r c} \right) \vec{n}_D(t) \right]_{t-r/c} \tag{32}$$

Here, the first term will, at low mach number, be the same as in the previous incompressible solution, but now with its effect delayed by the sound speed and the distance. The second term is the compressible acoustic contribution which has the same direction as the incompressible dipole, but with a strength depending on the rate of change of the incompressible dipole. In the following, the acoustic work done on the fluid by each isolated singularity will be computed first from a near field solution, and then compared with the far field Curle result. The retarded time behavior and the effect of multiple blades will be suppressed initially, and accounted for later, in order to emphasize the compressible behavior of these rotating singularities.

Blade Thickness Acoustics

It was shown previously that the power input to a rotating incompressible dipole that simulated blade thickness was zero, but now the acoustic power needs to be determined. As with the former case, certain simplifications from a distributed dipole model are possible since: 1.) potential flow solutions are additive, 2.) the motion of a non-lifting blade volume is well-described by a set of dipoles pointing toward the relative velocity, and 3.) from a distance

the precise shape of the surface is less important than the total displaced volume. With a restriction that the acoustic wavelength be large compared to the surface dimensions, it is thus possible to account for the effect of a moving non-lifting blade by considering a small set of dipoles. This set is to be distributed along the span, having strengths appropriate to the volume segments each represents and with direction opposing the local relative velocity. The thickness velocity potential at some distance from the several isolated sources is:

$$\phi_{a_T}(\vec{r},t) = -\vec{n}_o \cdot \sum_{i=1}^{N} \left[\left(\frac{D_{T_i}(t)}{4\pi r_i^2} + \frac{D_{T_i}'(t)}{4\pi r_i c} \right) \vec{n}_{D_i}(t) \right]_{t-r/c} \tag{33}$$

The acoustic behavior of this assembly can be characterized by examining any one component, each of which is equivalent to a simple sphere. Furthermore, the instantaneous velocity vector of each isolated volume can be considered as two superimposed phased linear oscillations in perpendicular directions in the plane of rotation. Consequently, for an observer in the x-z plane, at a distance from the source that is large compared to its size, Fig. 4, only the x direction motion will be sensed. Adapting Lamb's classic solution for the oscillation of a sphere at small amplitude about the origin, the periodic motion along the x axis can be defined as;

$$V_{x_i}(t) = V_{x_{0_i}} e^{j\Omega t} = \Omega R_i e^{j\Omega t} \tag{34}$$
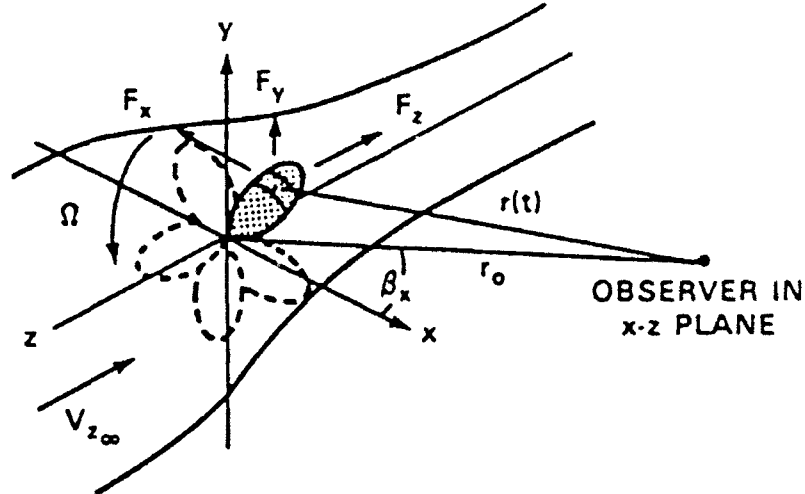

Fig. 4. Geometry for propagation of an acoustic signal
from a rotating F_x.

In order to satisfy a solid surface boundary condition at radius a_i, compatible with an isolated volume:

$$u_r \, (r=a_i) = \frac{\partial \phi_i}{\partial r} = V_{x_i} \cos \beta_x \tag{35}$$

Assuming that the strength of the dipole that simulates this sphere is also periodic allows the magnitude and phase to be computed from the boundary condition.

$$D_{T_i}(t) = \frac{3}{2} \Delta_i V_{x_{0_i}} \left(\frac{1-(ka_i)^2/2 - jka_i}{(1-(ka_i)^2/2)^2 + (ka_i)^2} \right) e^{j\Omega t} \tag{36}$$

The total pressure on the surface of the isolated sphere is given by the rate of change of the velocity potential.

$$p_{T_{T_{i_x}}} = -\rho \left(\frac{\partial \phi_i}{\partial t} \right)_{\Lambda = a_i} = \frac{\rho \cos\beta_x}{4\pi a_i^2} \frac{3}{2} \Delta_i \Omega^2 \overline{R}_i j (1+jka_i) D_{T_i}(t) \tag{37}$$

Because the kinetic energy of the fluid is symmetric over the surface of the sphere, it is possible to use this total pressure in a surface integral to find the force that the sphere applies to the fluid.

$$F_{a_{T_{i_x}}}(t) = \int_0^\pi p_{T_{T_{i_x}}} \cos\beta_x (2\pi a_i^2) \sin\beta_x d\beta_x = \frac{\rho\Delta_i}{2} \Omega^2 \overline{R}_i \left(j + \frac{(ka_i)^3}{2} \right) e^{j\Omega t} \tag{38}$$

The x direction force is seen to be primarily inertial, as in the incompressible case, but with a small real part. Taking the product of this force with the velocity of the body gives the real acoustic power introduced to the fluid.

$$\overline{P}_{a_i} = \frac{1}{2} \text{Re} \left\{ F_{x_i} \cdot V_{x_i}^* \right\} = \rho \frac{\Delta_i}{2} V_{x_i} \Omega \left(\frac{V_{x_i}}{2} \right) \frac{(ka_i)^3}{2} \tag{39}$$

Reviewing this calculation, it appears that any work that enters the fluid from the motion of a rigid surface must be transmitted via the added mass part of the dipole velocity potential. At first glance this does not seem consistent with Lighthill's formula for the force associated with the motion of a rigid body;

$$\overline{F}(t) = \frac{3}{2} \rho \Delta \frac{d\overline{V}}{dt} \tag{40}$$

which indicates that the fluid experiences an external force due to both the added and displaced masses. Turning to the Curle expression for the acoustic pressure in the far field due to motion of a sphere in the x direction,

$$p_{a_{T_{i_x}}} = \frac{\overline{n}_0 \cdot}{4\pi r c} \left[\frac{d\overline{F}_{T_{i_x}}}{dt} \right]_{t-r/c} \tag{41}$$

the proper value for the force in this equation is the total experienced by the fluid, without regard to any phase relationship. Using Lighthill's value;

$$P_{a_{T_{i_x}}} = \frac{\cos\beta_x}{4\pi rc} \left(\frac{3}{2} \rho\Delta_i\Omega^3\overline{R_i} \right) e^{j\Omega(t-r/c)} \tag{42}$$

which implies again that both the added and displaced masses are important.

Looking at the acoustic velocity potential in the far-field, it can be seen that the residual particle velocity is solely radial and in-phase with the acoustic pressure.

$$u_{r_{a_{i_x}}}(\overline{r}, t) = \frac{\partial\phi_{a_{i_x}}}{\partial r} = \frac{1}{\rho c} p_{a_{i_x}}(\overline{r}, t) \tag{43}$$

Combining these into an acoustic intensity and integrating to determine the average power being transferred across a large spherical surface by the x direction motion gives:

$$\overline{P}_{a_{T_{i_x}}} = \frac{\rho\Delta_i}{2} V_{x_i}\Omega \left(\frac{V_{x_i}}{2} \right) \frac{(ka_i)^3}{2} \tag{44}$$

This is identical to the power input from the surface force on the sphere oscillating in the x direction. Thus both the Lamb and the Curle expressions give the same result, but the former is accurate both near and far from the body.

In summary, the acoustic work associated with the motion of a rigid body is non-zero. It originates as a compressible effect of the sequence of impulses necessary to change the momentum of the surface vortex loops, i.e., the added mass, from one instant to the next. When both x and y motions are combined, this "thickness" source is seen to be a dipole of constant strength that rotates uniformly in direction with a total acoustic power output twice the above value. As shown in Fig. 5, the acoustic pressure field takes the form of outwardly spiralling positive and negative wave fronts propagating from the origin. These fronts also have a lobed pattern with a peak in the x-y plane and a null on the axis of rotation. This first order approximation is valid for low speed machines where the acoustic wavelength is large in comparison to blade dimensions.
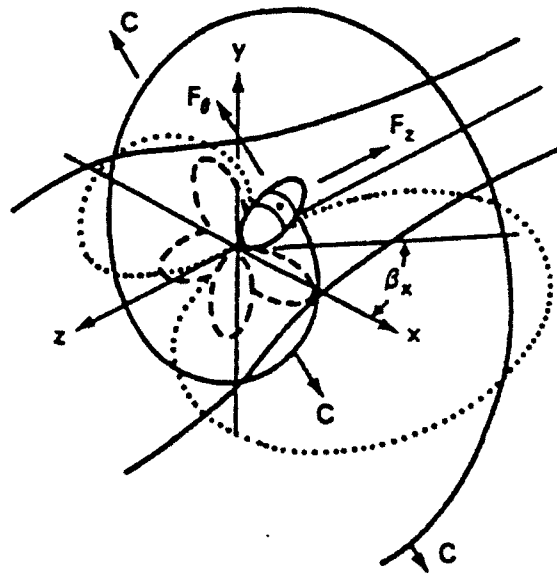

Fig. 5. Outwardly propagating spiral acoustic front for a rotating steady dipole.

Blade Lift Acoustics

In order to examine the acoustic effect of the lift vortex system, a blade of zero thickness, but having a trailing edge Kutta condition, will be considered. As pointed out previously, the boundary conditions for this geometry can be met by using the growing vortex loop singularity where the bound segment of the loop is attached to the blade. To simplify the analysis, the velocity of an isolated blade element will be taken again as a phased combination of linear oscillations in the x and y coordinate directions. Since the field response is cylindrically symmetric, the location of the observation point is arbitrary, and is taken to be in the x-z plane. When viewed from this point only x direction effects will be seen, and the projection of the length of an element of blade span will appear to vary periodically as:

$$(R_o - R_1)_{i_x}(t) = \overline{i} \, h_{o_i} \, j \, e^{j\Omega t} \tag{45}$$

Once a small vortex loop is formed, it only convects downstream at constant strength with no external force available to change the magnitude or direction and thus has no effect on the acoustics. However at the surface of the blade there exists an external periodic force needed to generate new vortex loop area.

$$F_{L_{x_i}}(t) = \rho G'_{L_{x_i}}(j\Omega) = \rho \Gamma_i V_{z_{D_i}} h_{o_i} \, j \, e^{j\Omega t} \tag{46}$$

Using this periodic form, it is straight-forward to write an expression for G, and therefore for the component of the velocity potential that originates at each instant using Equation (32).

$$\phi_{L_{x_i}}(r,\beta_x,t) = -\cos\beta_x \, \frac{\Gamma_i V_{z_{D_i}} h_{o_i}}{\Omega} \left[\left(\frac{1}{4\pi r_i^2} + \frac{j\Omega}{4\pi r_i c} \right) e^{j\Omega t} \right]_{t-r/c} \tag{47}$$

Examination of the near-field acoustic behavior for the periodic vortex loop is not as simple as was the case for the isolated sphere. Nevertheless it is still possible to compute the rate of doing acoustic work on the fluid external to an arbitrary sphere surrounding the source. Considering that the convecting dipoles passing through the surface of this sphere are described already by the incompressible work and not of interest here, the remaining average flux of real work passing radially through the control surface is;

$$\Phi_{L_{x_i}}(r, \beta_x, t) = \frac{1}{2} \, \text{Re} \left\{ P_{T_{L_{x_i}}} \cdot u^*_{r_{L_{x_i}}} \right\} \tag{48}$$

The total pressure and radial velocity functions can be formed easily from the periodic lift velocity potential by assuming that the time delay function does not alter the previous steady value, and by taking the variation of the position of the origin to be second order, thus giving:

$$P_{T_{L_{x_i}}}(r,\beta_x,t) = \cos\beta_x \, \rho \Gamma_i V_{z_{D_i}} h_{o_i} \left(\frac{j}{4\pi r^2} - \frac{\Omega}{4\pi r c} \right) e^{j\Omega(t-r/c)}$$

$$u_{r_{L_{x_i}}}(r,\beta_x,t) = \cos\beta_x \, \Gamma_i V_{z_{D_i}} h_{o_i} \left(\frac{2}{4\pi r^2 \Omega} - \frac{\Omega}{4\pi r c^2} + \frac{2j}{4\pi r^2 c} \right) e^{j\Omega(t-r/c)} \tag{49}$$

Integrating the power transfer over the surface of a sphere surrounding this source gives;

$$\overline{P}_{a_{L_{x_i}}} = \int_o^\pi |\Phi_{L_{x_i}}| \; 2\pi r^2 \; \sin\beta_x d\beta_x = \frac{F_{L_{x_i}}}{3} \left(\frac{F_{L_{x_i}}}{\rho}\right) \frac{\Omega^2}{8\pi c^3} \tag{50}$$

Expressing the magnitude of the lift force for any blade element in terms of the local lift coefficient and relative velocity provides an engineering level expression for the acoustic power.

$$\overline{P}_{a_{L_{x_i}}} = (F_{L_{x_i}} \overline{V}_{\theta_i}) \frac{C_{L_i} A_{B_i}}{48\pi R_i^2} \left(\frac{V_{z_{D_i}}}{c} \cdot \frac{W_i}{c} \cdot \frac{\overline{V}_{\theta_i}}{c}\right) \tag{51}$$

The first term represents the steady incompressible power produced by that blade element, so that the fraction of the lift power that appears as acoustic energy propagating outward is approximately:

$$\eta_{a_{L_x}} \approx \frac{1}{2} \frac{C_L}{24} \frac{A_B}{\pi R^2} (M_{z_D} \; M_w \; M_\theta) \tag{52}$$

Even though this is an averaged result, it can be seen that those radial elements with the largest product of lift coefficient and blade area will contribute most to the sound power.

Using the Curle formula to compute the far-field pressure and acoustic power at some point in the x-z plane gives exactly the above result for oscillation in the x direction. But a single application of the Curle equation does not reveal that the source is in fact a rotating dipole. This behavior can be seen from a computation for which the arbitrary observation point is located in any plane that includes the z axis. This shows that the total radiated acoustic signal from blade lift is due to a dipole of constant amplitude but with rotating direction and that the mean power is twice that from the x motion alone. As with the thickness sound, Fig. 5, the above result is correct when the Mach number is low and the acoustic wavelength is large compared to the blade dimensions.

Effect of Acoustic Delay

The acoustic signals from both the thickness and lift dipoles of a single blade have the form of outwardly propagating spiral pressure surfaces alternating in sign with radial distance from the origin. Although this model represents the general nature of the acoustic field for a single source at the origin, it is now important to account for the fact that the source is also rotating at some finite distance from the origin. Two changes occur. First, the time delay operation of the Curle equation provides a Doppler shift between locations where the source is perceived to be advancing or retreating. This distorts the fundamental pure tone in a manner related to the Mach number of the source. Secondly, for a balanced multi-bladed rotor, the sum of the separate force vectors in the plane of rotation is always zero. Thus if all were applied together, no acoustic signal would be predicted unless the retarded position was considered.

Both effects are accounted for by an analysis developed first by Gutin[15] for the lift force, with the consequence that the result is often referred to as "Gutin Sound". This analysis assumes that the acoustic pressure at any axially symmetric remote field point will always be periodic at the rotational frequency and its harmonics.

$$P_a(r_0, \beta_z, t) = \sum_{n=-\infty}^{\infty} A_n(r_0, \beta_z) e^{jn\Omega t} \tag{53}$$

The unknown Fourier coefficients may be computed by accounting for the acoustic delay from each angular position of the rotating blade as described by Quandt[16]. By the geometry of a rotating blade, the distance from any source point to an arbitrary field point measured from the origin is:

$$r \approx r_0 \left(1 - \frac{R}{r_0} \sin\beta_z \cos\theta_s \right) \tag{54}$$

Accounting for the acoustic delay, the angular position of the blade at the instant a signal originated that reaches the observer at some later time can be found from:

$$[\theta_s] = \Omega \left(t - \frac{r_0}{c} + \frac{R \sin\beta_z}{c} \cos[\theta_s] \right) + \theta_{s_0} \tag{55}$$

Solving for the Fourier reference angle appropriate to any field point gives

$$\Omega t = \left([\theta_s] + \frac{\Omega r_0}{c} - \xi \cos[\theta_s] - \theta_{s_0} \right) \tag{56}$$

where the parameter $\xi = \overline{V}_\theta \sin\beta_z / c$. This reference time can be substituted into the periodic form of the Curle equation to give an expression for the Fourier coefficients

$$A_n(r_0, \beta_z) = \frac{F \sin\gamma \sin\beta_z}{4\pi r_0 c} \frac{1}{2\pi} e^{jn(\Omega r_0/c - \theta_{s_0})} \frac{\partial}{\partial t} \int_{-\pi}^{\pi} \sin\theta_s e^{-jn(\theta_s - \cos\theta_s)} d\theta_s \tag{57}$$

Using the identity

$$e^{jn\xi\cos\theta_s} = \sum_{m=-\infty}^{\infty} (-j)^m e^{jm\theta} J_m(-n\xi) \tag{58}$$

and interchanging the order of integration and summation gives two Bessel functions that can be combined to give the acoustic pressure for a single source rotating about the origin at radius R. If there are B equal and equally spaced blades, and the blade force is assumed to be located at some radius, the resulting acoustic pressure becomes

$$P_{a_B}(r_0, \beta_z, t) = \frac{BF \sin\gamma}{4\pi r_0 R} \sum_{\ell=-\infty}^{\infty} B\ell (j)^{B\ell+1} J_{B\ell}(B\ell\xi) e^{jB\ell\Omega(t - r_0/c)} \tag{59}$$

For turbomachines of low blade number and low Mach number it is convenient to use the following approximate expression for the magnitude of the Bessel function

$$\left| J_{B\ell}(B\ell\xi) \right| \approx \frac{(B\ell\xi/2)^{B\ell}}{B\ell!} \; ; \quad B\ell\xi < 1 \tag{60}$$

If there is only one blade, $B = 1$, and the acoustic pressures for the first several harmonics are;

$$n = 1 \qquad p_{a_{B=1}}(r_o, \beta_z) = \frac{F \sin\gamma}{4\pi r_o R} (M_\theta \sin\beta_z) \cdot \frac{1}{2}$$

$$n = 2 \qquad p_{a_{B=1}}(r_o, \beta_z) = \frac{F \sin\gamma}{4\pi r_o R} (M_\theta \sin\beta_z)^2$$

$$n = 3 \qquad p_{a_{B=1}}(r_o, \beta_z) = \frac{F \sin\gamma}{4\pi r_o R} (M_\theta \sin\beta_z)^3 \cdot \frac{27}{16} \tag{61}$$

Thus for a rotor with one blade at low Mach number most of the acoustic power is in the first harmonic such that a somewhat pure tone is observed with a directional peak in the plane of rotation. If now there are three equal blades, the first three harmonics become:

$$n = 1 \qquad p_{a_{B=3}}(r_o, \beta_z) = \frac{3F \sin\gamma}{4\pi r_o R} (M_\theta \sin\beta_z)^3 \frac{9}{16}$$

$$n = 2 \qquad p_{a_{B=3}}(r_o, \beta_z) = \frac{3F \sin\gamma}{4\pi r_o R} (M_\theta \sin\beta_z)^6 \, 9^2 \cdot \frac{3}{40}$$

$$n = 3 \qquad p_{a_{B=3}}(r_o, \beta_z) = \frac{3F \sin\gamma}{4\pi r_o R} (M_\theta \sin\beta_z)^9 \frac{9^4}{560} \cdot \left(\frac{9}{8}\right)^4 \tag{62}$$

With multiple blades, summation of the vector forces causes considerable cancellation of the perceived signal and a further concentration of acoustic power at the blade passing frequency. In addition, the directionality is focused even more in the plane of rotation.

Ratio of Lift to Thickness Sound

The above results are independent of whether the source is due to the thickness or lift effect, since only the magnitude of the force applied to the fluid in the plane of rotation was important. The relative acoustic significance of the two sources can be assessed by taking the ratio of the total power from each for the single blade case.

$$\frac{\overline{P}_L}{\overline{P}_T} = \frac{F_L \nabla_\theta \frac{C_L}{24} \frac{A_B}{\pi R^2} (M_{z_D} M_w M_\theta)}{\rho \Delta_B \nabla_\theta^2 \, \Omega \, (ka)^3 \, (1/4)} \tag{63}$$

Defining a mean thickness for any isolated radial slice of the blade gives

$$A_B \overline{t} = \Delta_B \tag{64}$$

and the power ratio simplifies to

$$\frac{\overline{P_L}}{\overline{P_T}} = \left(\frac{C_L}{3}\right)^2 \left(\frac{\overline{R}}{\overline{t}}\right)^2 \left(\frac{V_{z_D}}{V_\theta}\right) \left(\frac{W}{V_\theta}\right)^3 \tag{65}$$

Since the lift coefficient and the velocity ratios are usually of the order of one, the ratio of lift to thickness sound power will vary as the square of the radius to thickness ratio. Near the blade tip the sound will originate mainly from the lift force, while at the root, thickness will be more important. Considering some nominal conditions for moderately loaded slender blades with a mean thickness to radius ratio of 5%, the pure potential Gutin sould power due to thickness may be 3% of that due to lift. For lightly loaded blades the proportion of the radiated sound power due to thickness can be significantly greater. In either case, the acoustic signal will be highly tonal, predominantly in the plane of rotation, and exhibit a high degree of cancellation from multiple equal and equally spaced blades.

CONCLUSIONS

Using the principles of unsteady potential flow, the energy transfer mechanisms of an ideal turbomachine have been explored using the example of a low-speed propeller. Both dipole and vortex loop singularities, which are normally considered to be distributed over the surface of a blade to match thickness and lift boundary conditions, were separated and treated as a series of elements summed in radial slices and rotating uniformly in the absolute frame. The resulting unsteady potential fields included both incompressible and acoustic energy terms, with corresponding power flows in both the near and far fields.

For the incompressible case, each vortex loop on each blade contributes to the work in the convected wake through application of the unsteady Bernoulli/Euler Equations. The torque on the shaft is represented conveniently by Lighthill's vorticity moment formula for the force on each moving body. Examination of the rotating dipole showed no contribution to the work, but did reveal how the added mass term is related to the surface impulse feature of potential flow. These vector impulses, associated with the creation of each flow field instantaneously from rest, provide a common link between both singularities.

For the slightly compressible case, both the thickness and lift characteristics of a rotating blade were shown to generate tonal acoustic power in the fluid. Each source appears as a steady rotating dipole with power input proportional to a real or apparent incompressible power, but reduced by the cube of the blade Mach number. For multiple blades, significant cancellation occurs in the far-field such that the perceived power is further reduced exponentially in Mach number. For typical blade loadings and geometries the lift effect is the dominant source.

Performing this unsteady potential analysis is useful in providing a complete and consistent description of the energy transfer processes of a turbomachine. In addition, it underscores the relationship between the dipole and vortex loop singularity functions and the surface impulses. This clarifies why these blade singularities, formed only to meet the geometric boundary conditions of the continuity equation, also satisfy inherently the momentum and energy requirements throughout the fluid field.

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1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE March 1993	3. REPORT TYPE AND DATES COVERED Progress

4. TITLE AND SUBTITLE
Unsteady Vortex Loop/Dipole Theory Applied to the Work and Acoustics of an Ideal Low Speed Propeller

5 FUNDING NUMBERS

6. AUTHOR(S)
Earl Quandt

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Surface Warfare Center
Code 27
Annapolis, MD 21402

8. PERFORMING ORGANIZATION REPORT NUMBER
CDNSWC/PAS–92/52

9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING /MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION /AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The vortex loop/dipole singularities used to simulate blade lift and thickness in potential flow models of turbomachines are examined in an unsteady form. The three-dimensional potential fields created as these singularities rotate in the absolute frame are shown to provide the input forces, ideal work and wake energy for an incompressible fluid, as well as the tonal acoustics for slight compressibility. For typical low Mach number propellers the sound field is dominated by the lift force and exhibits large cancellation effects with multiple blades. It is concluded that the unsteady formulation illustrates well the physics of a turbomachine, and in addition, highlights some basic principles of potential flow.

14. SUBJECT TERMS
Energy conversion; Power sources, Turbomachines physics; Acoustics, Fluid mechanics

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	

NSN 7540-01-280-5500

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